# Chapter 3

## Airfoil Aerodynamics

# 1.Airfoil Geometry and Physical Origin of Lift and Drag

## 1.1.Definition

An airfoil is the cross-sectional shape of a wing. Any slice of a wing cut parallel to the flight direction is an airfoil

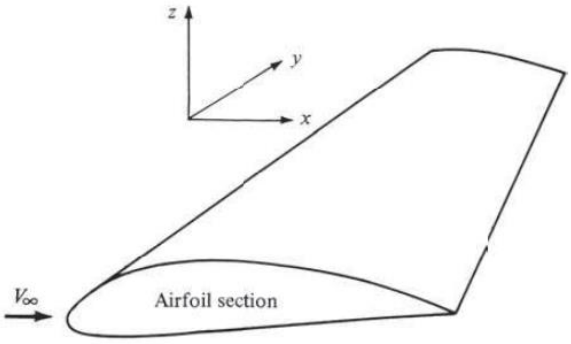



*Figure 1.An airfoil.*

## 1.2.Main Geometric Parameters

Look at Figure 2 as you read:

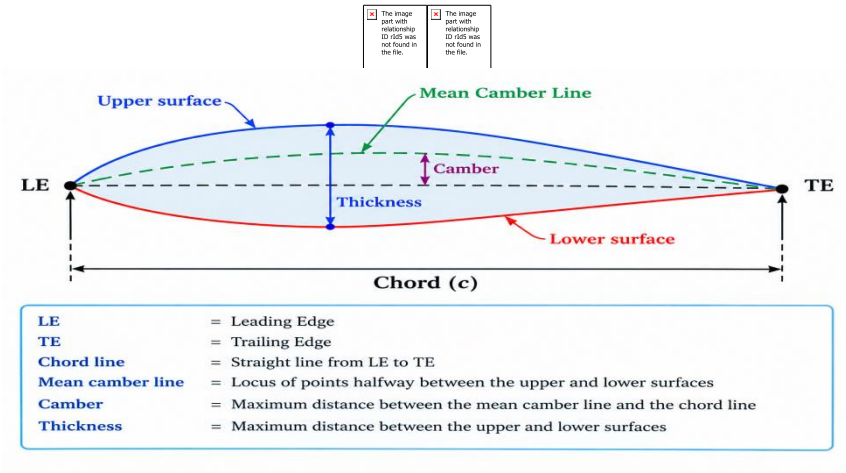



*Figure 2.Airfoil Geometry.*

*Table 1.Airfoil Geometry definition*

| Term | Definition | Symbol |
|---|---|---|
| Chord | Straight-line distance from the leading edge to the trailing edge | c |
| Chord line | Straight line connecting the leading edge and trailing edge | — |
| Mean camber line | Locus of points halfway between the upper and lower surfaces | — |
| Camber | Maximum distance between the mean camber line and the chord line | — |
| Thickness | Maximum distance between the upper and lower surfaces | t |
| Leading edge (LE) | Most forward point | — |
| Trailing edge (TE) | Most rearward point | — |
| Angle of attack | Angle between the chord line and the relative wind | α |

### 1.2.1.Chord Line

The chord line is the straight line connecting the leading edge and trailing edge. It is the reference line for measuring:

- Angle of attack α
- Camber
- Thickness

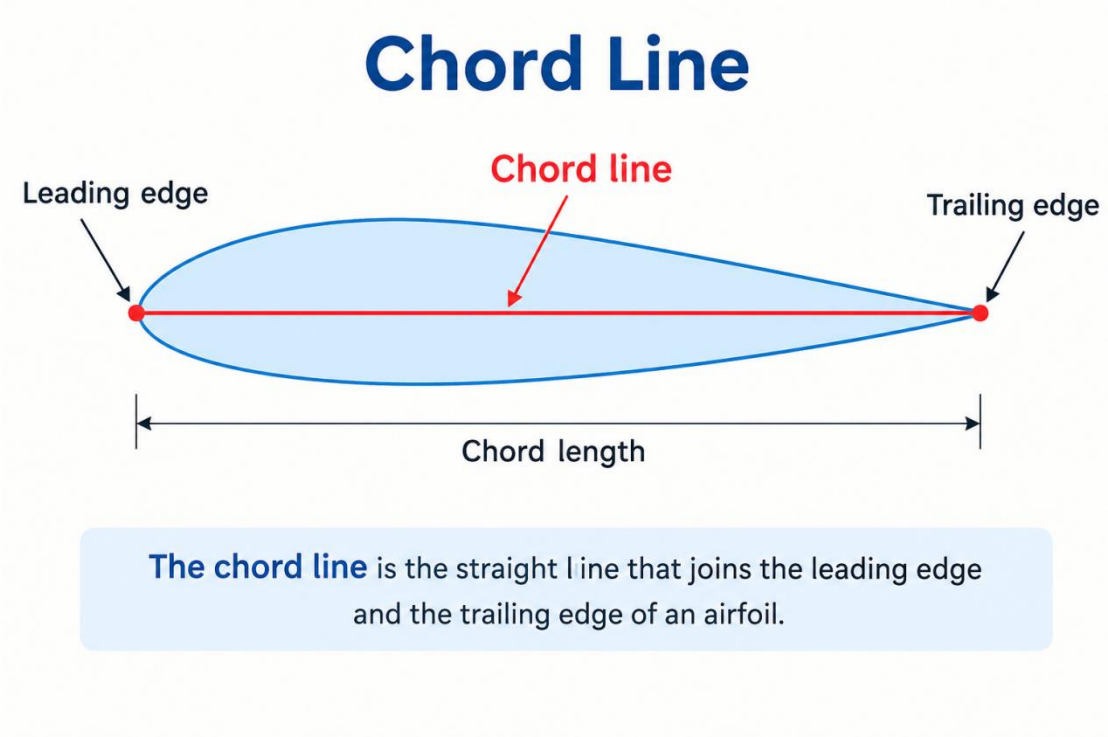



*Figure 3.Chord line.*

**Why Is the Chord Important?**

1. Reference length for all aerodynamic coefficients:

**a**.Lift coefficient:

A dimensionless measure of the lift produced by an airfoil:$C_l=\frac{L'}{q_\infty c}$

**b**.Drag coefficient

A dimensionless measure of the drag produced by an airfoil:$C_d=\frac{D'}{q_\infty c}$

**c**.Moment coefficient

A dimensionless measure of the pitching moment about a reference point:$m=\frac{M'}{q_\infty c^2}$

2. Scales the Reynolds number:The chord determines whether the boundary layer is laminar or turbulent. $Re_c=\frac{\rho_\infty V_\infty c}{\mu_\infty}$

3.Determines the moment arm for the center of pressure and aerodynamic center.

4.Normalizes geometric ratios:

Camber ratio: camber/$c$

Thickness ratio: t/c

Center of pressure location: $x_{cp}$/c

### 1.2.2.Camber

**Camber** is the maximum distance between the mean camber line and the chord line, measured perpendicular to the chord line

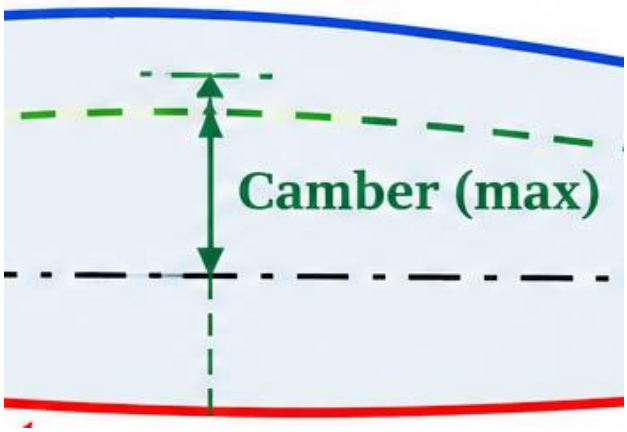



*Figure 4.Camber.*

#### 1.2.2.1.Types of Camber

- **Positive camber**: mean camber line lies above the chord line (typical for most airfoils).
- **Negative camber**: mean camber line lies below the chord line (used on some racing car wings).
- **Zero camber (symmetric airfoil)** : mean camber line coincides with the chord line.

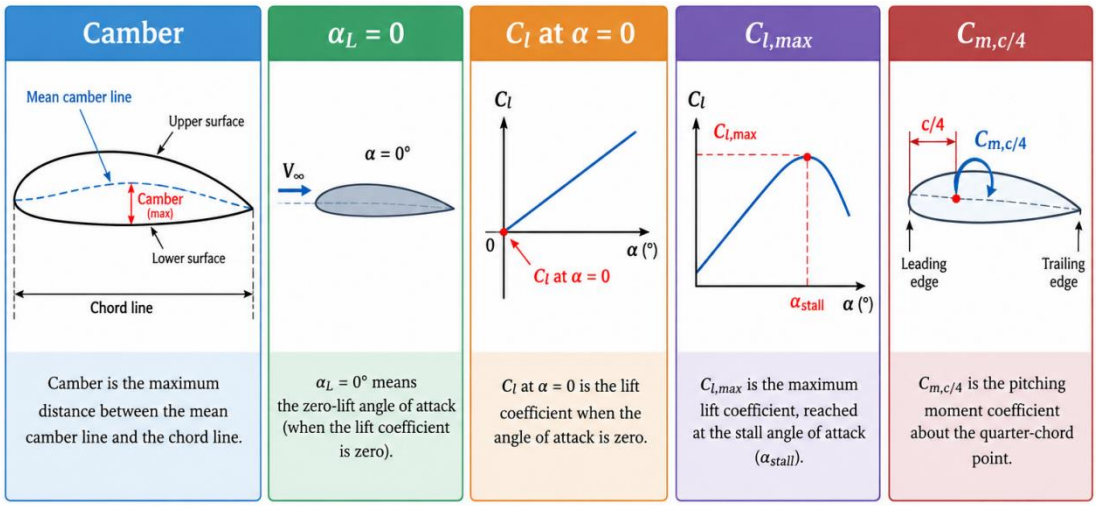



*Figure 5.Effect of Camber.*

**$\alpha_l$=0**:The **zero-lift angle of attack**: the angle at which the lift coefficient is ($C_l$=0). For a cambered airfoil, it is usually negative.

$C_l$ at α = 0:The **lift coefficient when the angle of attack is zero**. A cambered airfoil generally produces positive lift at α = 0°.

($C_{l,\max}$ ):The **maximum lift coefficient** that an airfoil can achieve before stall occurs. It is reached approximately at the **stall angle** ($\alpha_{stall}$).

($C_{m,c/4}$):The **pitching-moment coefficient about the quarter-chord point**, located at **25% of the chord from the leading edge**. For a conventional cambered airfoil, it is usually approximately constant in the linear lift range.

*Table 2.Effect of Camber on Aerodynamics*

| Camber | $\alpha_L$=0 | $C_l$ at α=0 | $C_{l,max}$ | $C_{m,c/4}$ |
|---|---|---|---|---|
| Zero (symmetric) | 0° | 0 | Lower | 0 |
| Positive (2%) | −2° | Positive | Higher | Negative |
| High (6%) | −6° | Large positive | Highest | Most negative |

## Formula for Zero-Lift Angle (Thin Airfoil Theory)

$$\alpha_{L=0} = -\frac{1}{\pi}\int_0^{\pi}\frac{dx}{dz}(\cos\theta_0 - 1)d\theta_0 \qquad \text{(III.1)}$$

## Why Camber Matters

Camber allows the airfoil to produce lift at zero angle of attack.This is useful for:Cruise flight (reduces fuselage angle, reducing drag).Improving $C_{l,max}$ for landing and takeoff.
Too much camber increases drag and pitching moment.

### 1.2.3.Incidence (Angle of Attack α)

**Definition:** The angle between the chord line and the relative wind (freestream velocity V∞) .

**Sign convention:**

**Positive α:** leading edge up relative to freestream

**Negative α:** leading edge down

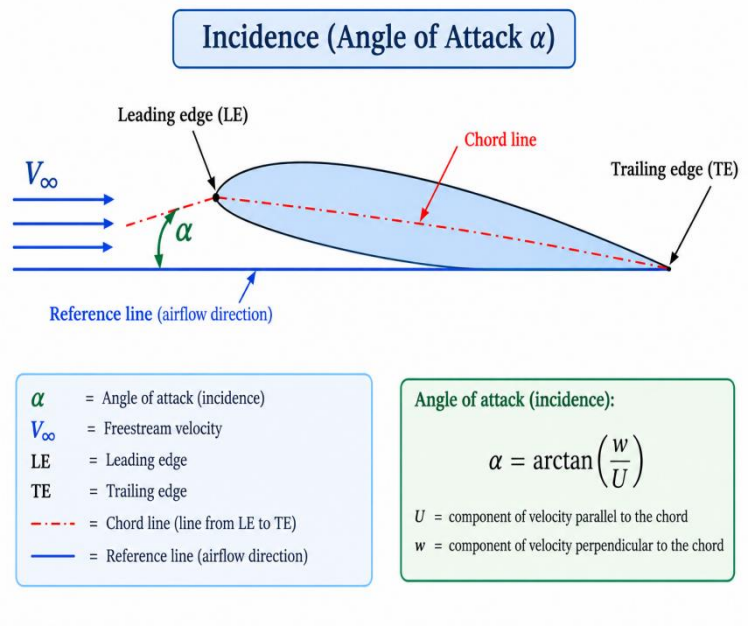



*Figure 6.Angle of attack.*

**Importance:**

Primary control variable for lift.

Cl increases linearly with α until stall.

At $\alpha=\alpha_L=0$: zero lift.
At $\alpha=\alpha$stall: $C_{l,max}$, then lift drops.

**Effective angle of attack (finite wing):**

$$\alpha_{eff} = \alpha - \alpha_i \qquad \textbf{(III.2)}$$

Where:

- α = geometric angle of attack
- $\alpha i$= induced angle of attack
- $\alpha eff$= effective angle of attack

### 1.2.4.Thickness (Complementary Parameter)

**Definition:** Maximum distance between upper and lower surfaces, perpendicular to the chord line.

**Thickness ratio:** t/c

*Table 3.Typical values*

| Aircraft type | t/c |
|---|---|
| Low-speed general aviation | 12–15% |
| High-speed subsonic transport | 8–12% |
| Supersonic fighter | 3–6% |

## 2. Lift and Drag: Physical Origin with Equations

### 2.1 Fundamental Sources of Aerodynamic Force

The aerodynamic forces and moments on a body are due to only two basic sources :

**1.Pressure distribution** p(s) over the body surface

**2.Shear stress distribution** τ(s) over the body surface

The normal and axial forces per unit span are obtained by integrating these distributions :

$$\boldsymbol{N' = -\int_{LE}^{TE}(p_u \cos\theta + \tau_u \sin\theta), ds_u + \int_{LE}^{TE}(p_l \cos\theta - \tau_l \sin\theta), ds_l} \quad \textbf{(III.3)}$$

$$\boldsymbol{A' = \int_{LE}^{TE}(-p_u \sin\theta + \tau_u \cos\theta), ds_u + \int_{LE}^{TE}(p_l \sin\theta + \tau_l \cos\theta), ds_l} \quad \textbf{(III.4)}$$

- $N'$: normal force per unit span.
- $A'$: axial force per unit span.
- $p_u, p_l$: pressure on the upper and lower surfaces.
- $\tau_u, \tau_l$: shear stress on the upper and lower surfaces.

- $\theta$: local surface angle.
- $ds_u, ds_l$: infinitesimal surface-length elements.
- $LE$: leading edge.
- $TE$: trailing edge.

### 2.2 Lift and Drag Definitions

**Lift** is the component of the resultant force perpendicular to V∞:

$$\boldsymbol{L = N\cos\alpha - A\sin\alpha} \quad \textbf{(III.5)}$$

**Drag** is the component of the resultant force parallel to V∞:

$$\boldsymbol{D = N\sin\alpha + A\cos\alpha} \quad \textbf{(III.6)}$$

- $L$: Lift force — perpendicular to the free-stream velocity V∞.
- $D$: Drag force — parallel to the free-stream velocity V∞.
- $N$: Normal force — perpendicular to the chord line.
- $A$: Axial force — parallel to the chord line.
- $\alpha$: Angle of attack.
- V∞ : Free-stream velocity.

### 2.3 Lift Coefficient and Drag Coefficient

**Lift coefficient** :

$$\boldsymbol{C_L = \frac{L}{q_\infty S}} \quad \textbf{(III.6)}$$

$$\boldsymbol{q_\infty = \frac{1}{2}\rho_\infty V_\infty^2} \quad \textbf{(III.7)}$$

**Drag coefficient** :

$$\boldsymbol{C_D = \frac{D}{q_\infty S}} \quad \textbf{(III.8)}$$

For a two-dimensional airfoil, lowercase coefficients are used:

$$c_d = \frac{D'}{q_\infty c} \quad \textbf{(III.9)}$$

## 2.4 Pressure Coefficient

The pressure coefficient indicates how much the local static pressure differs from the freestream static pressure, expressed relative to the freestream dynamic pressure.

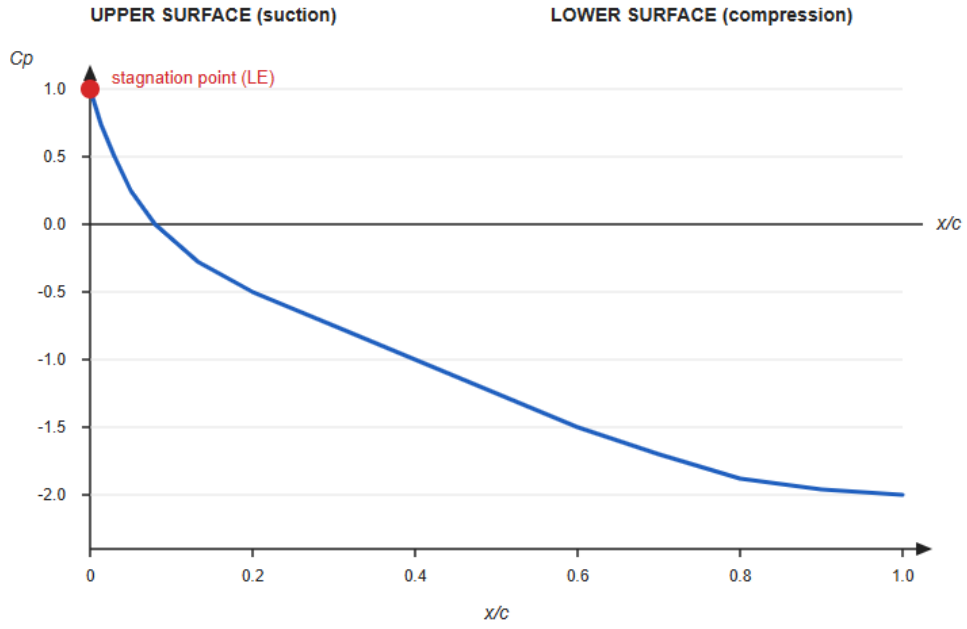



*Figure 7 .The pressure coefficient.*

The pressure coefficient is defined as :

$$C_p = \frac{p - p_\infty}{q_\infty} \quad \textbf{(III.10)}$$

For incompressible flow, using Bernoulli's equation (:

$$C_p = 1 - \left(\frac{V}{V_\infty}\right)^2 \quad \textbf{(III.11)}$$

For linearized supersonic flow :

$$C_p = \frac{2\theta}{\sqrt{M_\infty^2 - 1}} \quad \textbf{(III.12)}$$

where θ is the local surface inclination.

For newtonian hypersonic flow :

$$C_p = 2\sin^2\theta \quad \textbf{(III.13)}$$

## 2.5 Kutta-Joukowski Theorem

The lift per unit span is related to circulation by :

$$\boldsymbol{L' = \rho_\infty V_\infty \Gamma} \quad \textbf{(III.14)}$$

For a general two-dimensional body, this theorem states :

$$\boldsymbol{L' = \rho_\infty V_\infty \Gamma}$$

where Γ is evaluated around a closed curve enclosing the body.

## 2.6 Thin Airfoil Theory Results

For a symmetric airfoil :

$$\boldsymbol{\frac{dc_l}{d\alpha} = 2\pi} \quad \textbf{(III.15)}$$

Moment coefficient about the quarter-chord

$$c_{m,c/4} = 0$$

Lift coefficient for a cambered airfoil

$$\boldsymbol{c_l = 2\pi\left[\alpha + \frac{1}{\pi}\int_0^\pi \frac{dz}{dx}(\cos\theta_0 - 1)\, d\theta_0\right]} \quad \textbf{(III.16)}$$

Lift coefficient using the zero-lift angle

$$\boldsymbol{c_l = 2\pi(\alpha - \alpha_{L=0})} \quad \textbf{(III.17)}$$

Moment coefficient about the quarter-chord for a cambered airfoil

$$\boldsymbol{c_{m,c/4} = \frac{\pi}{4}(A_2 - A_1)} \quad \textbf{(III.18)}$$

where $A_1$ and $A_2$ are Fourier coefficients

- $z(x)$: vertical coordinate of the mean camber line.

· $\theta_0$: angular coordinate used in thin airfoil theory.

· $c_{m,c/4}$: pitching-moment coefficient about the quarter-chord point.

· ($A_1$,$A_2$): Fourier coefficients describing the camber-line slope distribution.

Important: The result $c_{m,c/4}$=0applies to an ideal symmetric thin airfoil in thin airfoil theory. For a cambered airfoil, the quarter-chord moment is generally nonzero.

## 2.7 Induced Drag

For a finite wing, the induced drag per unit span is :

$$D_i' = L' \sin \alpha_i \quad \textbf{(III.19)}$$

For small αi

$$D_i' \approx L' \alpha_i \quad \textbf{(III.20)}$$

The total induced drag is :

$$D_i = \rho_\infty V_\infty \int_{-b/2}^{b/2} \Gamma(y) \alpha_i(y)\, dy \quad \textbf{(III.21)}$$

The induced drag coefficient is :

$$C_{D,i} = \frac{2}{V_\infty S} \int_{-b/2}^{b/2} \Gamma(y) \alpha_i(y)\, dy \quad \textbf{(III.22)}$$

For an elliptical lift distribution :

$$C_{D,i} = \frac{C_L^2}{\pi AR} \quad \textbf{(III.23)}$$

For a general lift distribution :

$$C_{D,i} = \frac{C_L^2}{\pi AR}(1 + \delta) \quad \textbf{(III.24)}$$

Definition of the correction factor $\delta$

$$\delta = \sum_{n=2}^{N} n \left(\frac{A_n}{A_1}\right)^2 \quad \textbf{(III.25)}$$

Defining the span efficiency factor e=$(1+\delta)^{-1}$:

$$C_{D,i} = \frac{C_L^2}{\pi e A R} \quad \textbf{(III.26)}$$

### 2.8 Wave Drag

For a supersonic flat plate :

$$c_d = \frac{4\alpha^2}{\sqrt{M_\infty^2 - 1}} \quad \textbf{(III.27)}$$

For a hypersonic flat plate :

$$c_d = 2\sin^3\alpha \quad \textbf{(III.28)}$$

For small α:

$$c_d \approx 2\alpha^3$$

### 2.9 Total Drag on a Finite Wing

The total drag coefficient is :

$$C_D = c_d + C_{D,i} \quad \textbf{(III.29)}$$

where $C_d$ is the profile drag coefficient (from airfoil data) and $C_{D,I}$ is the induced drag coefficient.

For a finite wing :

$$C_D = c_d + \frac{C_L^2}{\pi e A R} \quad \textbf{(III.30)}$$

## 3. Aerodynamic Polars ($C_l$, $C_d$ as Functions of Angle of Attack)

### 3.1 Lift Coefficient vs. Angle of Attack

Thin symmetric airfoil :

$$\boldsymbol{c_l = 2\pi\alpha} \qquad (\alpha in radians)$$

Lift-curve slope (per radian and per degree)

$$\boldsymbol{\frac{dc_l}{d\alpha} = 2\pi \approx 0.11 per degree}$$

More precisely, $2\pi$ per radian corresponds to approximately 0.1097 per degree.

Cambered airfoil :

$$\boldsymbol{c_l = 2\pi(\alpha - \alpha_{L=0})} \quad \textbf{(III.31)}$$

Finite wing :

$$\boldsymbol{a = \frac{dc_l}{d\alpha} = \frac{a_0}{1+\frac{a_0}{\pi AR}(1+\tau)}} \quad \textbf{(III.32)}$$

where $a0=2\pi$ for thin airfoils, and τ is a function of the Fourier coefficients (typically 0.05 to 0.25).

For an elliptic wing:

$$\boldsymbol{a = \frac{a_0}{1+\frac{a_0}{\pi AR}}} \quad \textbf{(III.33)}$$

For a thin airfoil, ($a_0=2\pi$), so the equation becomes:

$$\boldsymbol{a = \frac{2\pi}{1+\frac{2}{AR}}} \quad \textbf{(III.34)}$$

### 3.2.Maximum lift coefficient

The maximum lift coefficient is not given by one universal formula; it depends on the airfoil shape, Reynolds number, Mach number, and stall characteristics.

$$c_{l,\max} = f(airfoil shape, Re, M)$$

$c_{l,\max}$ depends on airfoil shape, Reynolds number, and Mach number

### 3.3 Drag Coefficient vs. Angle of Attack

For an airfoil, the **profile drag coefficient** $c_d$ is a function of α and Re. Typical values:

- At low α: cd≈0.004 to 0.006
- At high α: cd increases due to flow separation

For a finite wing, the total drag coefficient is:

$$C_D = c_d + \frac{C_L^2}{\pi e AR} \qquad \textbf{(III.35)}$$

### 3.4 Drag Polar

A plot of CD vs. CL is called a drag polar.

For a finite wing :

$$\boldsymbol{C_D = c_d + \frac{C_L^2}{\pi e AR}} \textbf{ (III.36)}$$

**For a supersonic flat plate :**

$$\boldsymbol{C_l = \frac{4\alpha}{\sqrt{M_\infty^2 - 1}}} \textbf{ (III.37)}$$

Drag coefficient for a supersonic flat plate

$$\boldsymbol{C_d = \frac{4\alpha^2}{\sqrt{M_\infty^2 - 1}}} \textbf{ (III.38)}$$

Supersonic flat plate — eliminating ($\alpha$)

$$\boldsymbol{C_d = \frac{C_l^2 \sqrt{M_\infty^2 - 1}}{4}} \qquad \textbf{(III.39)}$$

Hypersonic flat plate — lift coefficient

$$C_l = 2\sin^2\alpha\cos\alpha \quad \textbf{(III.40)}$$

Hypersonic flat plate — drag coefficient

$$C_d = 2\sin^3\alpha \quad \textbf{(III.41)}$$

For small α:

$$C_l \approx 2\alpha^2$$

Drag coefficient

$$C_d \approx 2\alpha^3$$

Eliminating $\alpha$

$$C_d \approx \frac{C_l^{3/2}}{\sqrt{2}} \quad \textbf{(III.42)}$$

### 3.5. Lift-to-Drag Ratio

For a hypersonic flat plate with friction included :

$$\frac{C_l}{C_d} = \frac{2\alpha^2}{2\alpha^3 + C_{d,0}} \textbf{(III.43)}$$

Maximizing with respect to α:

$$\alpha_{(L/D)_{max}} = (C_{d,0})^{1/3} \textbf{(III.44)}$$

Maximum lift-to-drag ratio

$$\left(\frac{L}{D}\right)_{max} = \frac{0.67}{(C_{d,0})^{1/3}} \textbf{(III.45)}$$

Wave drag coefficient at this condition

$$C_{d,w} = 2C_{d,0}$$

### 3.6. Compressibility Effects

Prandtl-Glauert rule :

$$C_p = \frac{C_{p,0}}{\sqrt{1-M_\infty^2}} \quad \textbf{(III.46)}$$

$$C_l = \frac{C_{l,0}}{\sqrt{1-M_\infty^2}} \textbf{(III.47)}$$

$$C_m = \frac{C_{m,0}}{\sqrt{1-M_\infty^2}} \textbf{(III.48)}$$

Karman-Tsien rule :

$$C_p = \frac{C_{p,0}}{\sqrt{1-M_\infty^2}+\frac{M_\infty^2}{1+\sqrt{1-M_\infty^2}}\frac{C_{p,0}}{2}} \quad \textbf{(III.49)}$$

Laitone's rule :

$$C_p = \frac{C_{p,0}}{\sqrt{1-M_\infty^2}+\frac{M_\infty^2[1+(\gamma-1)M_\infty^2]}{2(1+\sqrt{1-M_\infty^2})}C_{p,0}} \quad \textbf{(III.50)}$$

Critical Mach number :

$$C_{p,cr} = \frac{2}{\gamma M_{cr}^2}\left[\left(\frac{1+\frac{\gamma-1}{2}M_{cr}^2}{1+\frac{\gamma-1}{2}}\right)^{\frac{\gamma}{\gamma-1}} - 1\right] \quad \textbf{(III.51)}$$

**Drag-divergence Mach number**: The freestream Mach number at which a large rise in drag coefficient occurs .

## 4. Ground Effect and Wingtip Effect

### 4.1 Wingtip Effect (Finite Wing Effects)

#### 4.1.1 Physical Mechanism

The pressure on the bottom surface of a lifting wing is higher than on the top surface. Near the wing tips, the flow curls around the tips from the high-pressure region underneath to the low-pressure region on top .

This creates wing-tip vortices that trail downstream . These vortices induce a downwash w at the wing .

#### 4.1.2 Downwash and Induced Angle of Attack

The downwash at spanwise location y0 is :

$$w(y_0) = -\frac{1}{4\pi}\int_{-b/2}^{b/2}\frac{d\Gamma}{dy}\frac{dy}{y_0-y} \qquad \textbf{(III.52)}$$

The induced angle of attack is :

$$\alpha_i(y_0) = -\frac{w(y_0)}{V_\infty} \qquad \textbf{(III.53)}$$

For an elliptical lift distribution :

$$w = -\frac{\Gamma_0}{2b} \quad \textbf{(III.54)}$$

$$\alpha_i = \frac{\Gamma_0}{2bV_\infty} = \frac{C_L}{\pi AR} \quad \textbf{(III.55)}$$

#### 4.1.3 Induced Drag

The induced drag coefficient for an elliptical lift distribution :

$$C_{D,i} = \frac{C_L^2}{\pi AR} \quad \textbf{(III.56)}$$

For a general lift distribution :

$$C_{D,i} = \frac{C_L^2}{\pi e AR} \quad \textbf{(III.57)}$$

where e≤1 is the span efficiency factor.

### 4.1.4 Effect of Aspect Ratio

**Aspect ratio** :

$$AR = \frac{b^2}{S} \quad \textbf{(III.58)}$$

The induced drag coefficient is inversely proportional to AR . The primary design factor for minimizing induced drag is making the aspect ratio as large as possible .

## 4.1.5 Effect of Aspect Ratio on Lift Slope

For a finite wing :

$$a = \frac{a_0}{1+\frac{a_0}{\pi AR}(1+\tau)} \quad \textbf{(III.59)}$$

For an elliptic wing :

$$a = \frac{a_0}{1+\frac{a_0}{\pi AR}} \quad \textbf{(III.60)}$$

As AR→∞, $a \to a0$.

## 4.2 Ground Effect

ground effect can be derived from the finite-wing theory presented.

### 4.2.1 Physical Mechanism

When a wing operates near the ground:

The wing-tip vortices are constrained by the ground. The ground acts as a mirror surface, creating a mirror image vortex system below the surface.

The image vortices have opposite strength and are located at the mirror-image position below the ground.

The downwash at the wing is the sum of the contributions from the real trailing vortices and the image vortices.

The image vortices induce an **upwash** at the wing, partially cancelling the downwash from the real vortices.

### 4.2 .2Mathematical Formulation

For a single horseshoe vortex at height h above the ground, the downwash at the wing due to the real trailing vortices is :

$$\boldsymbol{w_{\text{real}} = -\frac{\Gamma}{4\pi h}} \quad \textbf{(III.61)}$$

The image vortex at depth h below the ground induces an upwash at the wing:

$$w_{\text{image}} = +\frac{\Gamma}{4\pi h} \quad \textbf{(III.62)}$$

The net downwash is:

$$w_{\text{net}} = w_{\text{real}} + w_{\text{image}} = 0 \quad \textbf{(III.63)}$$

In reality, the cancellation is not perfect for a finite wing, but the downwash is significantly reduced.

### 4.2.3 Effects on Aerodynamic Coefficients

The reduction in downwash leads to:

**Increased effective angle of attack**:

$$\alpha_{\text{eff}} = \alpha - \alpha_i$$

Since αi is reduced, αeff is increased.

Increased lift coefficient for the same geometric angle of attack:

$$C_L = a_0(\alpha_{\text{eff}} - \alpha_{L=0})$$

**Reduced induced drag** since $C_{D,i} \propto \alpha i$:

$$C_{D,i} = \frac{C_L^2}{\pi e AR}$$

The reduction in αi reduces the induced drag.

Increased lift-to-drag ratio:

$$\frac{L}{D} = \frac{C_L}{C_D}$$

## Practical Consequences

- Aircraft experience a "floating" sensation when landing or taking off.
- Ground effect is important for **WIG (Wing-In-Ground) craft**.
- The effect diminishes rapidly with altitude; it is typically significant only within one wingspan or less above the surface.